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FOR STATIC THICK ISOTROPIC BEAMS, A COMPARISON OF DIFFERENT DISPLACEMENT FIELDS

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Abstract: - A review of displacement and stress for static isotropic beams based on displacement fields is presented. Different displacement-based shear deformation theories' displacement fields were compared. In terms of thickness coordinates, the theories include parabolic, sinusoidal, hyperbolic, and exponential functions, as well as the transverse shear deformation effect. Governing differential equations and associated boundary conditions of the theory are derived by employing the static version of the principle of virtual work. The various displacement field depends on a parameter 'λ', whose value is determined to give results closest to each other. By comparing the current study to various available findings in the literature, the quality of the work can be determined.

Keywords: *Thick beam, Displacement fields, Static isotropic beam, Sshear deformation, Principle of virtual work.*

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I INTRODUCTION

The beam and plate theories are the active areas of research since the historical time. The classical engineering theory of beam bending due to Bernoulli and Euler dates to 1705 and had its origin in the first mathematical model of nature of the resistance of a beam developed by Galileo Galilei in 1638. Saint Venant in 1856 presented the complete solution of the beam problems considering bending and shear stresses. The classical theory of plate bending had its origin in the pioneering work of Sophie Germain carried out in 1815. The theory reached maturity due to the well-known Kirchhoff hypothesis and the resolution of famous boundary conditions paradox by Kirchhoff in 1850.

Thick beams and plates, either isotropic or anisotropic, basically form two- and three-dimensional problems of elasticity theory. Reduction of these problems to the corresponding one- and two-dimensional approximate problems for their analysis has always been the main objective of research workers. The shear deformations in beams and plates with the three-dimensional nature of these problems further intensified the research interest in their accurate analysis. As a result, numerous refined theories of beams and plates have been formulated in last three decades which approximate the three-dimensional solutions with reasonable accuracy.

Rankine [1], Bresse [2] were the first to include both the rotatory inertia and shear flexibility effects as refined dynamical effects in beam theory. This theory is, however, referred to as the Timoshenko beam theory as mentioned in the literature by Rebello, et al. [3] and based upon kinematics it is known as first-order shear deformation theory (FSDT). Rayleigh [4] included the rotatory inertia effect while later the effect of shear stiffness was added by Timoshenko [5]. Timoshenko showed that the effect of shear is much greater than that of rotatory inertia for transverse vibration of prismatic beams. The first correct boundary conditions for the Timoshenko beam were derived by Kruszewski [6] and Dengler and Goland [7] and further it was well discussed by Dym and Shames [8]. In Timoshenko beam theory transverse shear strain distribution is constant through the beam thickness and therefore requires shear correction factor to correct the strain energy of deformation. History of shear coefficient is given by Kaneko [9] and critically examined by Hutchinson and Zillmer [10], Hutchinson [11]. Further discussion on the shear coefficients in beam bending is presented by Rychter [12]. Stephen and Levinson [13] have introduced a refined theory incorporating shear curvature, transverse direct stress, and rotatory inertia effects. The governing differential equation is similar in form to the Timoshenko beam equation. However, the theory requires two coefficients, one for cross sectional warping and the second dependent on the transverse direct stresses. These coefficients for various cross sections are evaluated. Soler [14] developed the higher order theory for thick isotropic rectangular elastic beams using Legendre polynomials and Tsai and Soler [15] extended it to orthotropic beams. Effects of shear deformation and transverse normal stress are included. The interrelation between classical beam theory and higher order theories is explicitly brought out. The displacement field of the theory assumes the transverse normal component of deformation as quadratic in thickness coordinate and includes the effect of transverse shear deformation. The in-plane displacement is assumed to be the linear function of thickness coordinate.

II BEAM UNDER CONSIDERATION:

Consider a thick isotropic simply supported beam of length L in x direction, Width b in y direction and depth h as shown in Figure 1. Where x, y, z are Cartesian coordinates. The beam is subjected to transverse load of intensity q (x) per unit length of beam. The beam can have any boundary and loading conditions.

Assumptions made in the theoretical formulation:

1. The axial displacement (u) consist of two parts:

a. Displacement given by elementary theory of bending.

b. Displacement due to shear deformation, which is assumed to be parabolic, sinusoidal, hyperbolic and exponential functions in nature with respect to thickness coordinate.

2.The transverse displacement (w) in z direction is assumed to be function of x coordinate.

3.One dimensional constitutive law is used.

4.The beam is subjected to lateral load only

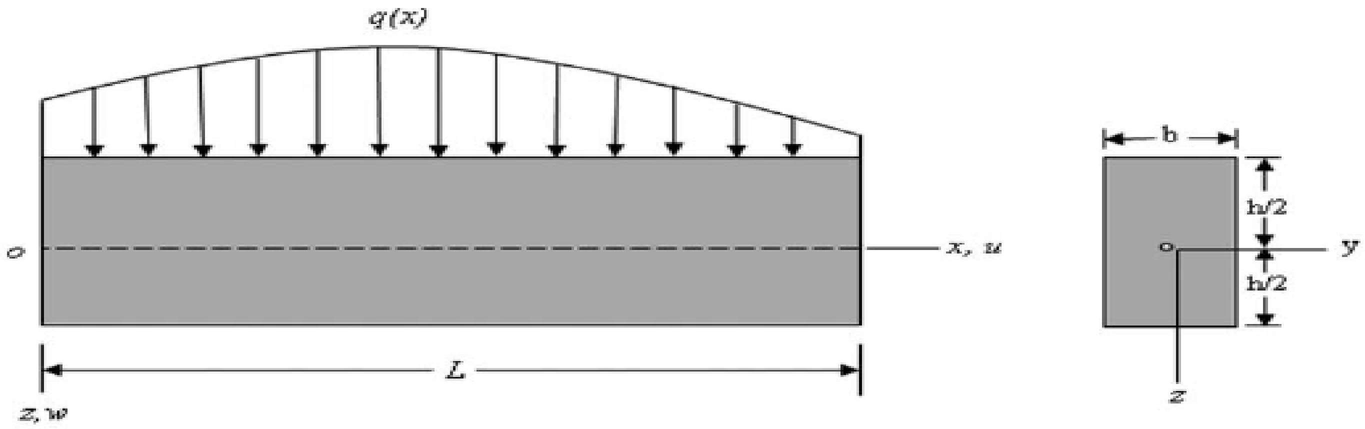

Figure 1: Beam bending under x-z plane

The Displacement Field:

Based on the above mentioned assumptions, the displacement field of the present beam theory can be expressed as follows:

$$u(x,z) = -z\frac{\partial w}{\partial x} + f(z)\phi(x) \qquad (1)$$

$$w(x,z) = w(x) \qquad (2)$$

Where, u = Axial displacement in x direction which is function of x and z.

w = Transverse displacement in z direction which is function of x and z.

φ = Rotation of cross section of beam at neutral axis due to shear which is an unknown function to be determined and it is a function of x.

f(z) = Function assigned according to the shearing stress distribution through the thickness of beam.

Normal Strain:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x} \qquad (3)$$

Shear Strain:

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = f'(z)\phi \qquad (4)$$

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The one dimensional Hooke's law is applied for isotropic material, stress бx is related to strain ε x and shear stress is related to shear strain by the following constitutive relations:

$$\sigma_x = E\varepsilon_x = E\left[-z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x}\right]$$

$$\tau_{zx} = G\gamma_{zx} = Gf'(z)\phi \qquad (5)$$

Where E and G are the elastic constants of material.

Governing differential equation and boundary conditions:

Governing differential equations and boundary conditions are obtained fromPrinciple of virtual work. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}(\sigma_x\delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx})\,dxdz$$

$$-\int_{x=0}^{x=L} q(x)\delta w dx = 0 \qquad (6)$$

Where δ = variational operator. Employing Green's theorem in above Equation successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$A_0\frac{\partial^4 w}{\partial x^4} - B_0\frac{\partial^3 \phi}{\partial x^3} = q(x) \qquad (7)$$

$$B_0\frac{\partial^3 w}{\partial x^3} - C_0\frac{\partial^2 \phi}{\partial x^2} = 0 \qquad (8)$$

$$-A_0\frac{\partial^3 w}{\partial x^3} - B_0\frac{\partial^2 \phi}{\partial x^2} = 0 \qquad (9)$$

The associated boundary conditions obtained are of following form:

Where w is prescribed

$$A_0\frac{\partial^2 w}{\partial x^2} - B_0\frac{\partial \phi}{\partial x} = 0 \qquad (10)$$

Where dw/dx is prescribed.

$$-B_0\frac{\partial^2 w}{\partial x^2} + C_0\frac{\partial \phi}{\partial x} = 0 \qquad (11)$$

Where φ is prescribed.

Where A0, B0, C0 and D0 are the stiffness coefficients given as follows

$$C_0 = E\int_{h/2}^{-h/2} f^2(z)dz \qquad D_0 = G\int_{h/2}^{-h/2} [f'(z)]^2 dz$$

The Displacement Fields of several theories and Stiffness coefficients for various Displacement Fields

Based on the literature reviewed in above section, the displacement fields of several layer beam theories available in the literature are compared in Table 1.

Governing differential equation and boundary conditions:

The governing equation for static flexure of beam can be obtained from equation 7 and 8. Using this equations, general solution for w(x) and φ(x) can be obtained. Final solution for w(x) and φ(x) can be obtained depending upon the loading and boundary conditions of the beams. Substituting the final solution for w(x) and φ(x) in displacement field, the final displacement can be obtained. Finally, the axial stress σx, can be obtained by using stress strain relationship as given in equation 5. The transverse shear stress τxz can be obtained by using the constitutive relation or by equilibrium equations.

Where

$$\lambda^2 = \frac{\beta}{\alpha}; \beta = \left(\frac{GAD_0}{DB_0}\right); \alpha = \left(\frac{C_0}{B_0} - B_0\right)$$

Table 1.The Displacement Fields of several theories and Stiffness coefficients for various Displacement Fields

Sr No	Theory	Year	Displacement Field f(z)	A_0	B_0	C_0	D_0	*Y For H/2*
1	Ambartsumian S. A	1958	$\frac{z}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right]$	0.083333333 Eh^3	0.008333333 Eh^5	0.000843333 Eh^7	0.000843333 Gh^5	17.2265/h
2	Kruszewski	1949	$\frac{5z}{4}\left[1 - \frac{4z^2}{3h^2}\right]$	0.083333333 Eh^3	0.008333333 Eh^3	0.084325396 Eh^3	0.833333333 Gh	17.97/h
3	Reddy and Krushnamurty	1984	$z\left[1 - \frac{4z^2}{3h^2}\right]$	0.083333333 Eh^3	0.066666666 Eh^3	0.053968253 Eh^3	0.533333333 Gh	18.2005/h
4	Y.M.Ghugal	2012	$\frac{(1+\upsilon)\,h^2}{4}\;z\left[1 - \frac{4z^2}{3h^2}\right]$	0.083333333 Eh^3	0.021666667 Eh^5	5.700396825 X10^3- Eh^7	0.056333333 G h^5	17.97/h
5	Arya	2000	$\sin\left(\frac{\pi z}{h}\right)$	0.083333333 Eh^3	0.0202642367 Eh^2	0.5000Eh	4.934802201Gh	16.20/h
6	Ray	2003	$\left[\frac{3z}{2h} - \frac{2z^3}{h^3}\right]$	0.083333333 Eh^3	0.1000 Eh^2	0.12428571 Eh	1.200Gb/h	17.97/h

7	Zenkor	2013	$\left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^3}\cosh\left(\frac{1}{2}\right)\right]$	0.08333333 3Eh^3	0.06664158 8 Eh^3	0.05392925 5 Eh^3	0.5329322 54Gbh	17.95/h
8	Daouadji	2013	$\frac{3\pi}{2}\left[h\tanh\left(\frac{z}{h}\right) - z\sec^2 h\left(\frac{1}{2}\right)\right]$	0.08333333 3Eh^3	0.06552765 3Eh^2	0.05223794 Eh^3	0.5156374 81Gbh	16.70/h
9	Bessaim	2013	$\left[z\cosh\left(\frac{\alpha}{2}\right) - \frac{h}{\alpha}\sinh\left(\frac{\alpha z}{h}\right)\right]$ $\alpha = \left(\frac{\pi}{4} - \frac{24}{\pi^3}\right)$	0.08333333 3Eh^3	1.07556718 8x10^-6Eh^2	1.405008x1 0^-11 Eh^3	1.38848x1 0^-10Gbh	17.96/h
10	Karama	2009	$ze^{-2(z/h)^2}$	0.08333333 3Eh^3	0.062273Eh 2	0.047368 Eh^3	0.468148G bh	14.70/h

III CONCLUSION:

The widespread use of shear flexible materials in aircraft, automobiles, shipbuilding, and other industries has sparked

interest in accurate beam structural behaviour prediction. The development of refined theories to obtain the correct structural behaviour resulted from the flexural analysis of thick beams.The primary goal of this review article is to present the various methods for analysing thick isotropic beams that are currently available, as well as to provide guidance to researchers for future research.Many displacements based higher order shear deformation theories have been reported in the literature for the flexural analysis of thick beam which are presented in Table 1. Based on the review presented in this paper, the value of λ obtained for various displacement fields is in the range of 17 to 18 and the displacement and stresses obtained by using this value of λ are gives results closest to each other. The accuracy of present work is ascertained by comparing it with various available results in the literature.

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